Monetary velocity in a systemic perspective

From Macroeconomics to Currency-Thinking

Jens Martignoni

Themes: Monetary Theory / Social and Complementary Monetary System

2nd EDITION 27.10.2015

Paper presented at the
1ST ANNUAL CONFERENCE OF MONETARY RESEARCH CENTER
MONETARY POLICIES IN THE BALKANS. THE FUTURE OF THE EURO AND EUROZONE IN THE BALKAN REGION
October 15-16, 2015, Sofia, Bulgaria

Abstract

The concept of a velocity of money also called velocity of circulation of money is a part of the Quantity Theory of Money. One intention has been to connect price based equilibrium theories (where money is de facto cancelled out) back to money. The idea was a variable velocity, which describes how fast the existing money (thought in pieces of gold or in bills) would circulate. To question it allows an insight into the paradigmatic base in economic thinking and leads to a different approach of the flow of money, which might help to better shape the task of money in todays economy.

The author tries to reconsider the "velocity-idea" by looking very carefully on the real money-flow-phenomena in a simple small-scale complementary currency. By changing from a market-centred-view to a money-centred-view it is the aim to focus on the construction of currency. The reciprocal quality of currency and the time bound qualities of payment are described and the money supply is defined. By a timeslice method the dynamic money-flow phenomena can be visualized. The results are then merged into the velocity-equation and discussed again.

This systemic approach allows a more accurate view on monetary flow phenomena of closed systems. Such a systemic approach could be elaborated and might open some new perspectives for the understanding or simulation of monetary-economies.

Author:
Jens Martignoni, research fellow at Cooperative Seminary of the University of Cologne, lecturer at Swiss Distance University of Applied Sciences - FFHS and director and head of research at FleXibles - Association for the investigation of new-economy systems, Zurich, Switzerland
jens.martignoni@uni-koeln.de

1. Introduction

In our times we became used to the fact that one special type of currency[1], which I would call the *state based private bank money* or in short *the existing money-system (tems)*, has succeeded in becoming the absolutely dominant or *monopoly money*. Weather it is called Dollar, Pound, Yen Swiss Franc or Lev it is briefly of the same construction. The Euro might be slightly different because it is supranational, but the differences are very small and moreover seem not to be in its favour as a stable and wealth-preserving instrument. So most of the time we talk about money, we think in fact only of one type of currency, the actual monopoly money and the features it has. We rarely are aware that we do not reflect on its construction. In fact *tems* is a bundled type consisting of at least three components: Coins or minted money, bills and book money. Other distinctions are possible and have been already suggested by economists[2]. Most of them try to describe existing topics of money or currencies, but rarely do economists care about the *construction of currency* respectively *the design criteria of a currency* and maybe some would protest on an approach like this, which tries to highlight some crucial features of construction. Rather do most existing theories about money take the technical details of currencies as a somehow intangible or historically developed prerequisite and start with questions about the market. Then money appears in this market as a medium of exchange, a store of value and a unit of account. In this sense money becomes a very fascinating phenomena popping out of history. This mystery has inspired hundreds of highly valuable scientists to write about it, but until today it often remained a veal, something neutral or even inconsiderable in economics.
Another main reason for that inexplicability might be the giant dimensions of the money-system. Many millions of users and therefore many billions of transactions every year had to be registered in society already at the beginning of economic science in the 18th century. From this sheer quantity and without the initial possibility of highly developed computing like today, reductionist methods had to be developed to get at least get a glimpse of all what was happening in money system. Further reasons of ignoring the currency design might be the trade or market centred view of most economists and the strong metallism or believe in money as a commodity in the past which continues misleading not only scientists but especially politicians until today. Nowadays the already highly developed money system became even bigger and more complicated and so the initial assumptions never had been reviewed successfully since. By reconsidering the above points the author tries to find out more about moneys background. Therefore the research was based on the following premises:

1. Reduce the size and complexity of the money-system to regain a better overview
2. Leave the market centred view and focus on currency-construction
3. Take a strictly nominal view of money

It happened that this had not to be a reductionist and abstract theoretical method but became already reality by the introduction of alternative money, also called complementary currency. By experiments made by practitians, many such currencies started as a greenfield development during the last decades without many preconceptions about money. In fact many of these experiments revealed some very interesting things about money exactly because they fill the above three points. To study such alternative currencies or to develop new models allows a much

[1] The terms *money* and *currency* were used sometimes quasi synonymously in this article. As far as possible or helpful the following distinction was made: The indication *money* was used as a more general or abstract description of the phenomenon. The term *currency* was used as a more specific technical or systemic description of practical forms of money.

[2] A compound-idea of the money system was suggested by Walter Eucken. He differed money as tangible goods (becoming coins later), delivery of goods (obligations) and credits by trusted authorities (banks, state) and saw todays money as a clipped compound, see Eucken, 1941, p.142-147

deeper insight in the basic ideas behind money. This was a starting point of the authors research about small monetary systems during the last years.
In this paper the author tries to reconsider the "velocity-idea" of money by looking very carefully to the real money-flow-phenomena in a small model-complementary-currency. To be successful in this, some maybe unorthodox basic steps in defining the systemic circumstances had to be taken. By that the article provides a scratch of a different view on money. It has not the claim to be fully developed and many important voices in the monetary discussion are not yet referenced. So its approach has to be developed further but still might be an impulse on how monetary theory could be improved with the help of studying small-scale currency-systems.

2. The velocity-idea

One task in the general approach of past economic science was the idea of kind of finding natural laws and mathematizing it in elementary formulas and terms. The idea was presumably strongly influenced by the very successful natural sciences, particularly physics. At the end of the nineteenth century the concept of a velocity of money, also called velocity of circulation of money was reintroduced by scholars. The idea was a variable velocity which should describe how fast the existing money (thought in pieces of gold or in bills) would circulate and which should serve as an indicator for fluctuating prices.

2.1 Preparatory steps

To prepare the further remarks we take first a look at the physical definitions of the velocity. The simplest form of velocity in physics, the rapidity of motion is more precisely called speed:

$$v = \frac{s}{t}$$

v speed in meter per seconds
s distance travelled in meter
t time used in seconds

Speed is a scalar quantity that refers to "how fast an object is moving." Velocity in contrast is a vector quantity that refers to "the rate at and the direction in which an object changes its position."

$$\vec{v} = \frac{\Delta \vec{s}}{\Delta t}$$

v velocity in meter per seconds (vector)
Δs displacement in meter (vector)
Δt time used in seconds

If we even take it more serious and get one step further to the instantaneous velocity:
We can express the instantaneous velocity of an object or a particle, at any particular time *t*, as the derivative of the position with respect to time:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

This is true if $v = f(x)$ is a function which is differentiable at every point. Just to remember that many functions are not differentiable:

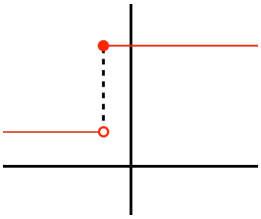

Figure 1: Jump Function[3]

The function in Figure 1,does not have a derivative at the marked point, as the function is not continuous there (it has a jump discontinuity). Later on we will reference differentiability in the case of moneys "velocity".

2.2 Quantity Theory of Money

Much of the first economists thinking was targeting markets, working and trade phenomena. Especially the price and value topics were carefully examined[4]. Money therefore was considered as a neutral mirror of values. But out of that it was difficult to explain the purchasing power of money and effects like inflation, when an expansion of the volume of money influences prices. So there had to be found a connection between prices and money to connect price based equilibrium theories (where money is de facto cancelled out) back to money. The first ideas of a quantity theory of money (QTM) were already discussed by Davanzati,(1588) and later Locke, Hume and Cantillon. All their findings were developed further in the 20th century by economists like Schumpeter and others and became todays "canonical form"[5] by Irving Fisher 1911[6]:

$$MV_T = PT$$

It is also called Fisher identity, because Fisher already mentioned that the equation might better be seen as an identity[7]

M total nominal account of money in circulation (money supply) in currency units
V_T velocity of money for all transaction in a given timeframe
P the price level
T the volume of transactions of goods and services

The velocity then comes out by transforming the equation as follows:

$$V_T = \frac{PT}{M}$$

The equation looks very simple but integrates some quite fancy considerations:

---

3 Graphics from Wikipedia, The Free Encyclopedia, https://en.wikipedia.org/w/index.php?title=Derivative&oldid=683407651 access at 09.10.15;22:00

4 e.G. by Adam Smith, 2007, p.26: *In order to investigate the principles which regulate the exchangeable value of commodities, I shall endeavour to show, First, what is the real measure of this exchangeable value; or, wherein consists the real price of all commodities, Secondly, what are the different parts of which this real price is com- posed or made up*.

5 see Paul, Axel T., 2012, p.114

6 Fisher, Irving, 1912, p.24

7 John Munro, Prof. at the Department of Economics, University of Toronto remarks: **This is more of an identity () or tautology than it is a causal equation**: it simply states that total spending, in terms of the money stock multiplied by the rate of its turnover or circulation, necessarily equals total spending in terms of the total volume of monetary transactions multiplied by the current price index. The two values on each side of the sign are necessarily identical https://www.economics.utoronto.ca/wwwfiles/archives/munro5/QUANTHR2.htm

The price level P for example is an enormously abstract value which had to be filtered out of the total amount of all transactions by dividing by something like an artificial "number of pieces sold" (T) which might become especially difficult when mixed product prices, service hours or packet-prices, etc. So it was obviously necessary to take the whole turnaround or the total nominal amount of transactions in currency units per period instead, e.g. the BIP of the country. This transforms the equation into:

Velocity of Circulation = Total Spending in a given timeframe (S_T) divided by Amount of Money in circulation (M)

$$V_T = \frac{S_T}{M}$$

This seems a meaningful equation but it has already lost its link to prices and therefore to values of goods and services. Instead we see S_T the aggregated transactions in money-units per time. So the velocity V_T becomes a measure for the *intensity of the use of money* or *use rate of money* instead, but remain a kind of fuzzy and indirect parameter because "the use of money" itself is not really defined but points back to the price and purchasing problem without really solving something.
So we arrive at a point where many other economists already had been and reflected on the inherent problems of the concepts of the quantity theory of money and the velocity of money. Here we will not go back further into the long history of the discussion but prepare for a different approach which will allow us to question the money supply and the velocity of this equation.

2.3 Taking a currency-centred view

As already mentioned: Usually economists take a market- or price-centred view to describe the economy and money and its role therein. The equilibrium theory e.g. tries to give an understanding of the whole economy using a "bottom-up" approach, starting with individual markets and agents.
If we like to understand the functioning of money these approaches are not very useful because it always brings in the topic of value by the price. At that point money has to be preconsidered to realize this prize, otherwise it would not be a real price. By realizing the price, money and value unite in a short moment. As interesting this might be, it is distracting from the construction of money behind. Value is a strongly fluctuating thing, tightly bound to human needs, thoughts or guesses. To avoid this difficult area a money-centred (or better currency-centred) view is proposed here. In fact, this is nothing new and since Adam Smith such a distinction was proposed as he remarked:
When, by any particular sum of money, we mean not only to express the amount of the metal pieces of which it is composed, but to include in its signification some obscure reference to the goods which can be had in exchange for them, the wealth or revenue which it in this case denotes is equal only to one of the two values which are thus intimated somewhat ambiguously by the same word, and to the latter more properly than to the former, to money's worth more properly than to the money.[8]
If we take Smith's words seriously and distinguish money (counting side) from money's value or purchasing power (value side), the following model is suggested:

8 Smith, Adam, 2007, p.224

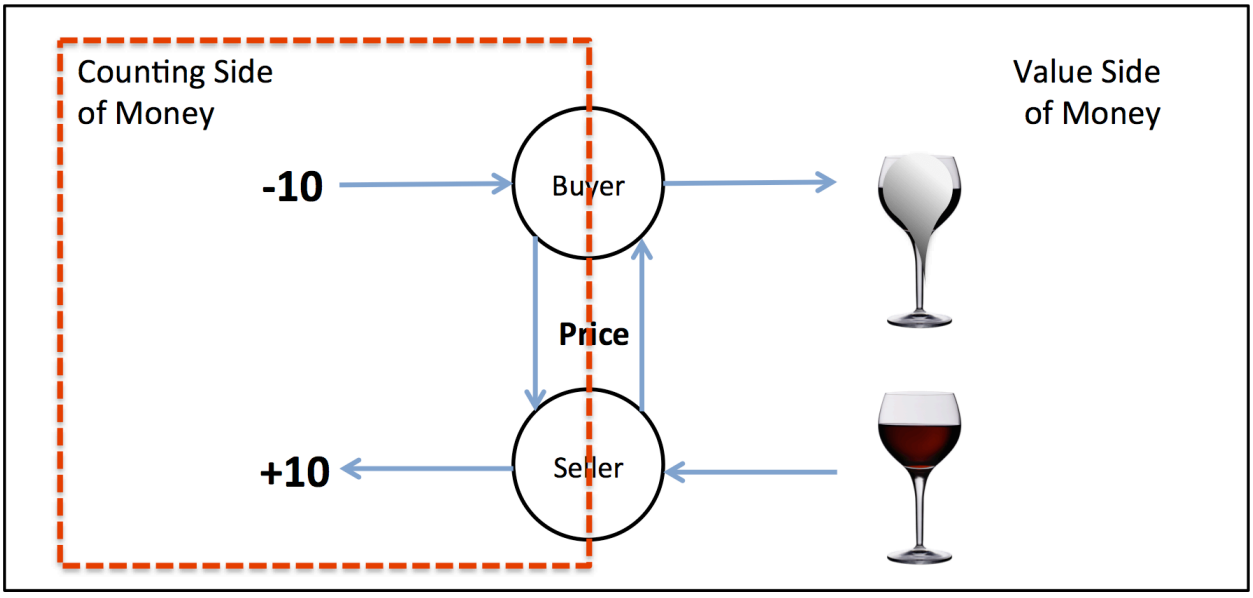

Figure 2: Counting side of money as separate system to be handled

A look to the value side is not the aim of this article. Instead we will investigate on the counting side, the "mechanical part" of money, which seems to be very simple at first sight, but is maybe not. This side seems to be only rigid and boring accounting but strangely enough it is not well recognized to be an optimal starting area to study money and its basic principles. The "independence" of this system is not yet further discussed, but is usually taken for granted by all accountants. How the two sides exactly are connected and which interconnecting operations exist, must be the topic of further research.
There has been such an approach of The Stock-Flow consistent model (SFC) of Copeland/Tobin that is very interesting and the idea discussed here comes close to it. The main difference is that SFC remains in the approach to explain the whole economy (value side) and use the counting side only as image of *tems*, without first question or discuss the rules of the currency used.

3. A Systemic approach

The advantage of a complementary currency concept is to be able to demonstrate the really systemic side of a currency because it can be defined from the bottom and it is very small compared to the real economies *tems*. By focusing on the construction of the system and by being aware that it might be better first to understand and adapt the characteristics of the system, we have a reversed view on economics: Currency therefore is seen as an operating system of economy. It has to be designed optimally to serve the economic processes and therefore should be stable, bug-free and tested as to use a analogy from the IT-world.

3.1 Currency as a closed system

If we could demonstrate that a specific complementary currency scheme could be seen as a closed system, the determination of such a thing as "velocity" must be much easier to do. Centralized models make it easier to state the unity or closeness of a currency-model but in fact it would be a interesting question if a non closed currency ever was intended and what it would look like.

To make it easy we start with a simple LETS[9]-model. LETS is usually a pure book money. There is a central instance ("bank") which is doing the bookkeeping but has not to be involved any further. The basic rules are:

- Every participant has exactly one account
- At the beginning everybody is starting at zero, with the possibility to overdraw to a certain credit-limit.
- In the most basic version, the credit-limit shall be the same for each account
- Every payment has to be registered or booked to become valid
- The sum of all accounts is always zero
- At the beginning there are no taxes or fees for payments

Such a system is closed. If a first payment comes, the sum is drawn from the buyers account and added to the sellers account. The buyer now is as much in the negative as the seller in the positive.
In a 12 participants economy based on a LETS scheme such a payment of 100 units from A (buyer) to G (seller) looks like the following:

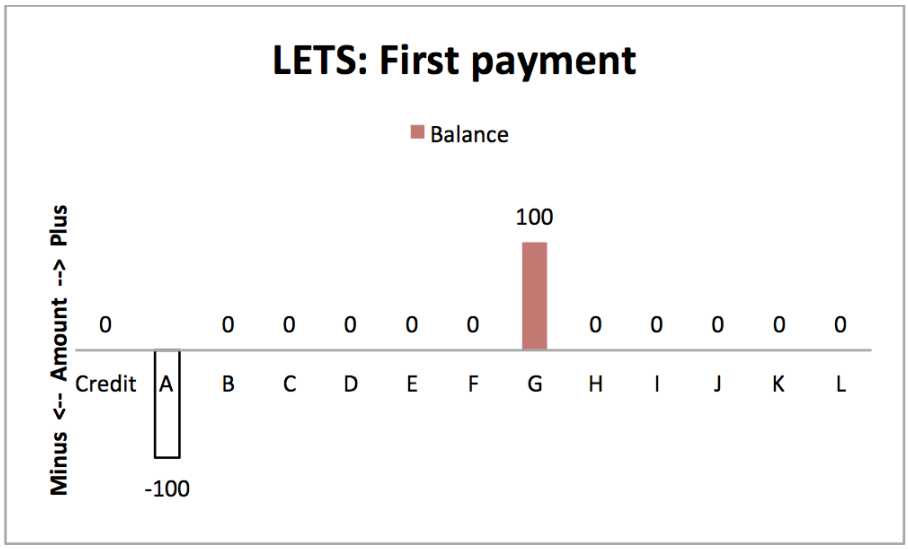

Figure 3: First payment in a 12 members LETS system

The simple accounting presented is really meant to be technically as in information technology (IT) the handling of bits and bytes. This is not fully in accordance with usual accounting structures[10] because it does not (yet) consider the value side of currency.

3.2 Reciprocal quality of currency

The "money" in above system appears as a symmetrical, reciprocal operation and in fact this is one basic feature of a currency, but is seldom recognized. It is true for every type of currency or money for the moment of a payment but also further like in debts and wealth. Here we confine ourselves to the payments only:

- One side has to give the money, meaning it is subtracted from its wallet, account, wealth or maybe future (in case of a cheque or bill)
- The other side has to receive the money, meaning is added to its wallet, account, wealth or store

[9] LETS: Local Exchange Trading System, developed by Michael Linton 1983 in Canada, see Wikipedia https://en.wikipedia.org/wiki/Local_exchange_trading_system, 05.10.15

[10] see Hughton Budd, Christopher, 2015, p.6

3.3 Time bound quality of payments

Another very important feature of money as a payment is time. The transfer of the payment from one to the other account in the LETS example will be quasi-simultaneous, only restricted by the computing velocity of the used PC. Subtraction at one account and addition at the other account are therefore assumed as instant (quasi timeless)[11] validation. The second important notion is: the time *when* this operation is done is crucial and has to be identified.
The immediate transfer is the ideal and systemically optimal version. In such a case of a closed system the total money supply is stable and remains constant. As soon as there would be time-gaps between payment and receive of payment (or vice versa in case of some types of credits), this would change the system and it would have to be examined if it could be still taken as a closed system.

To conclude it as a system principle:

Currency in a closed system appears in a strictly reciprocal operation between two parties when at a given time a certain number of units is subtracted at one side and added to the other side.

3.4 Currency supply (Money Supply)

Now we have a look at the currency supply of such a LETS-system where we have accounts with overdraw facility down to a specified credit limit. The maximum potential (i.e. money supply) of such a LETS model-system could be calculated easily by:

$$M_{\max} = \sum_{i=1}^{n} (A_B + C_L)_i$$

M_{max}: maximum possible money supply (units)
A_B: Account Balance (units)
C_L: Credit-Limit (units)
$A_B + C_L$ could also be seen as the *capacity* of the certain account

In our 12 member-system with a given credit limit of 100 units for all accounts, the calculation for the initial situation (all accounts starting at zero) would be as follows:

$$M_{\max} = \sum_{i=1}^{12} (0 + 100)_i = 1'200 units$$

This maximum money supply equals the maximum potential of the system for money transfer at a given time. This means the potential of transfer of the maximum sum of units (payment) per account, i.e. every owner of an account would spend the maximum amount he is allowed at the same time. If executed it might look like this:

[11] This was also set as a definition of cash flow by Stuetzel, see „Gleichzeitigkeit von Buchung und Gegenbuchung“, Stuetzel, W., 1978, p.57f.

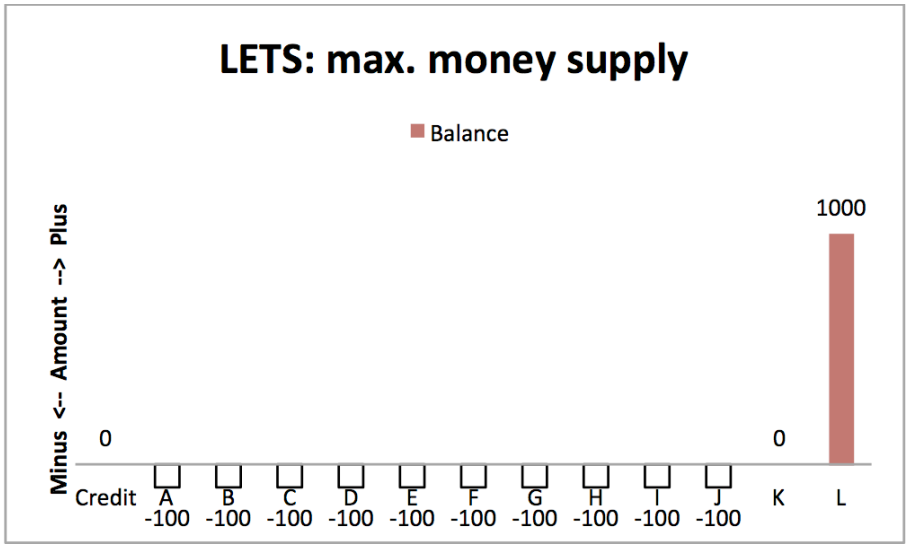

Figure 4: Example of maximum money supply executed in a 12 members LETS system

The maximum money supply could also be seen as *the maximum imbalance* of the system.

As we already defined payment as an operation which simultaneously and reciprocally subtracts a certain number of units from one account (buyer) and adds it to another account (seller), the operation *payment* could be formalized as follows:

$$p_i : (a_i : k_m \Rightarrow k_n)_{T_x}$$

p_i: defined payment (operation-identifier)
a_i: amount of transfer (number of units)
k_i: accounts between which the reciprocal operation is done
=>: Direction of transfer (buyer=>seller)
T_x: Time of execution of the payment

In the example shown in Figure 4, the array of operations for the execution at the given time T_x looks like the following:

p_i	a_i	k_m	k_n
1	100	A	L
2	100	B	L
3	100	C	L
4	100	D	L
5	100	E	L
6	100	F	L
7	100	G	L
8	100	H	L
9	100	I	L
10	100	J	L
11	100	K	L
12	100	L	K

Table 1: Example of maximum payments at a time in a 12 members LETS system

This was a look at the situation in a closed money-system at a certain time. At such a time it is possible to calculate a maximum money supply. It is not yet possible to talk about velocity,

because this needs an on-going time. But it is possible to see that the real amount of units used to pay at any time has to be lower than the maximum money supply at the same time:

$$(A)_{T_x} = \sum_{i=1}^{n} (a_i)_{T_x} \leq (M_{\max})_{T_x}$$

4. Dynamic money-flow phenomena

To talk about money-flow and maybe get our velocity out somewhere, we have to enhance the simple model. Until now we did not really consider that the payments take place at different times. Let's first take a simple version, when the payments are made one after another.

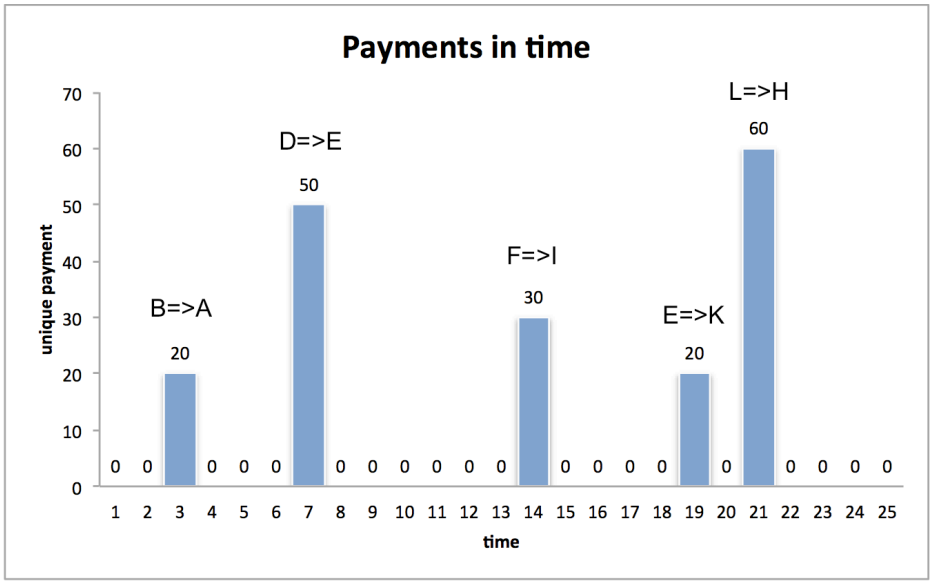

Figure 5: Example of 5 Payments in time in a 12 members LETS system

Obviously each payment is done at a certain time and needs a certain slice of time[12]. If two were at the same time, additional rules had to be set how the paying would be processed[13].
An important point is, that the validation of the payment shall be separated from the execution. The validation means the point where the buyer looses its ownership of the amount and the seller gains his. The method that is inherently present and is mostly used to guarantee the uniqueness of an amount of (especially electronic) money is basically the time-step or slice of time method. Usually we are used to a sequential, one after another payment like at the supermarkets cashier, but as more complex distributed systems like the clearing of banks or especially high frequency trade systems show, it needs a clocking to be able to determine valid bookings.
To demonstrate this, the payments of Figure 5 are taken first into a 5'clocks slice of time grid. The second additional feature is to show the accounts and their real changes. Every payment needs an oppositional operation on the two involved accounts. The plus at the sellers and the minus at the buyers, of the same amount of course.

[12] *Precision regarding time: A model of short-run determination of macro-economic activity necessarily refers to a slice of time. It is one step of a dynamic sequence, not a repetitive equilibrium into which the economy settles.* Tobin, J., 1981, p.13

[13] Take a situation where at the same time A has 100 credit and shall pay 150 to B and B has also 100 credit and shall pay 50 to A. Sequentially processed it would only work if first B pays, then A. Done by a *clearing* process only one booking oft he difference is made. So it is dependent on the rules (configuration) of the system, what will happen.

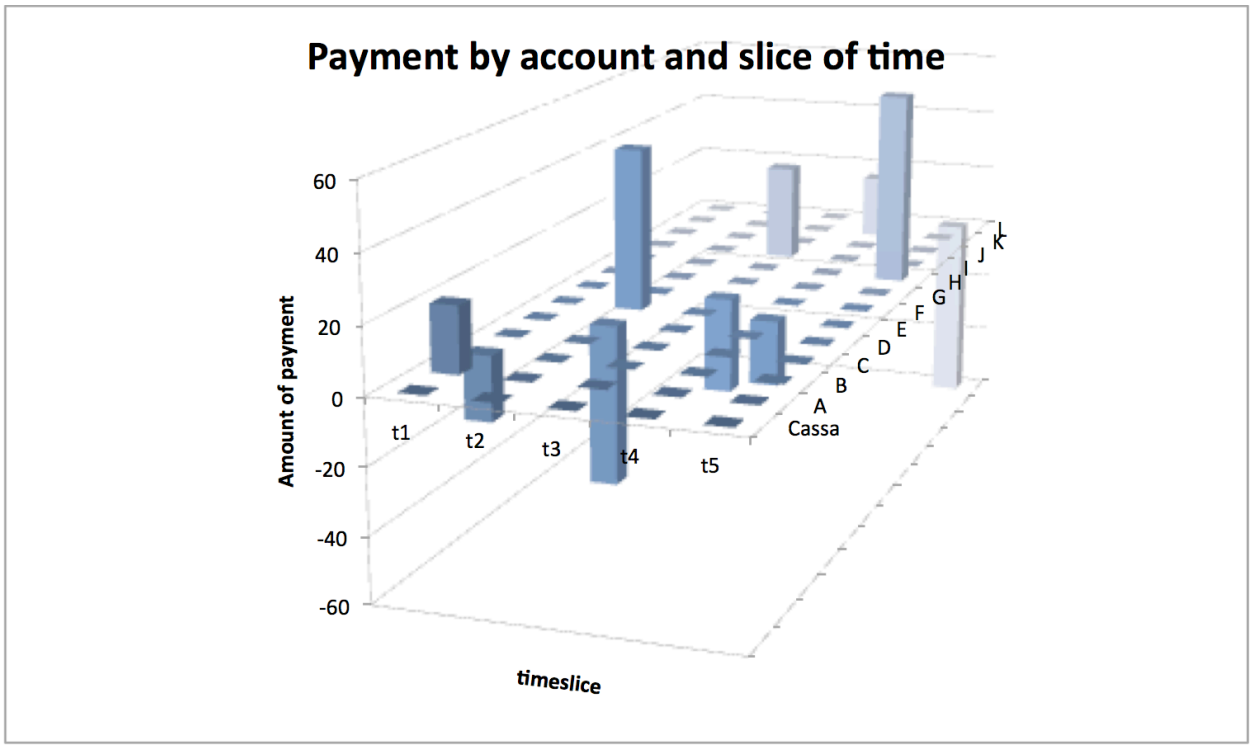

Figure 6: Payment by account and slice of time in a 12 members LETS system

Now we have a real picture of "the money-flow" and see that is not a flow at all but a sequence of impulse-like happenings in a matrix[14].
In the picture with the full balance of the accounts for each slice of time it is much more difficult to see the same changements even if all balances start at zero.

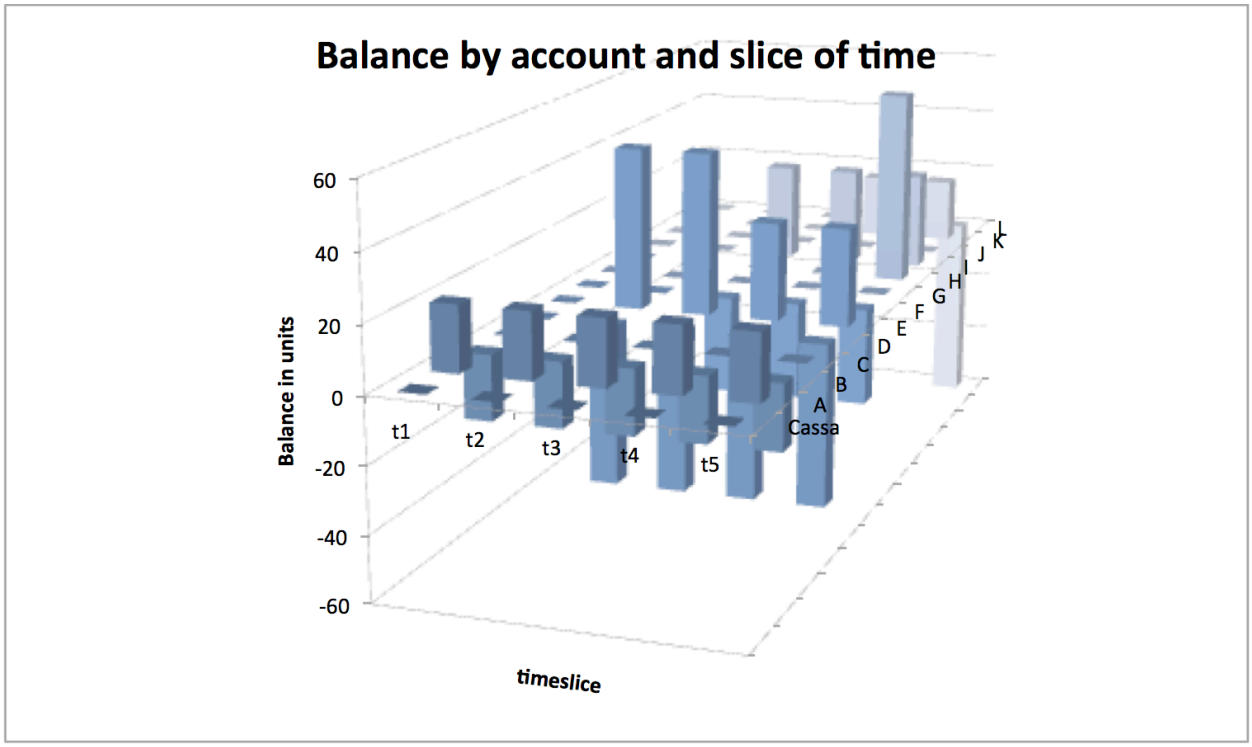

Figure 7: Balance by account and slice of time in a 12 members LETS system

[14] The blockchain technology uses such an approach to register all payments.

As we can clearly see in Figure 6 the payments occur as time bound "jumping" events. One certain payment is limited by the boundaries of the "buyers"-account. All payments of the same slice of time are limited by the "maximum money supply" of the same slice of time. If we would like to calculate the "monetary velocity" the same way it is done in QTM but respecting the slice of time, then we would have:

$$V_T = \frac{S_T}{M} = \sum_{T_x=1}^{T} \frac{(A)_{T_x}}{(M_{\max})_{T_x}}$$

5. Discussion

What about the "monetary velocity" after these steps? Isn't it still the same? The result might look the same at the end but two mayor differences were pointed out:

1. We demonstrated that we not have a continuous mathematical function of the payments that would be differentiable at every point, but we have a disruptive function which is not differentiable at any point.
2. We demonstrated that payments are timebound and could only be processed in certain time steps.
3. We demonstrated that payments are dependent on the buyers actual account balance.

It could be shown that the velocity does not resemble really to physical velocity and the variable V_T should better be called *intensity of use* or *use rate of money*.
Additionally we defined the maximum money supply as the potential of how much spending would be possible at a certain time. This comes close to the definition of M1 of the most central/national banks[15] but is much more basic and seems to be more accurate in its logic. This is true if not only our example complementary currency but also todays *tems*-money would be a closed system. Further it could be shown that there must be a connection between actual liquidity of money and the payments to calculate a meaningful "monetary velocity". That indicates that the use of M2 or even M3 is, against common use[16], not very advisable.

Questions which arise now:

- Are these findings really applicable to our existing money-system (*tems*)
- Is the *tems* a closed system and if not, how does this come together with the rules of bookkeeping and basic mathematics?
- The main focus of the treatise was on payments, but what about savings?
- What about the interaction to the effectively traded goods and services connected with the payments?

Final remarks:

- By a systemic view of a closed currency system the "velocity of money" could be examined much deeper and a more accurate view on monetary flow phenomena appears on a micro level.
- Such a systemic approach could be further elaborated and might open some new perspectives for the understanding or simulation of monetary-economies.

[15] e.g. Swiss National Bank; M1: *Defined as currency in circulation (bank notes/coins), sight deposits at banks, postal accounts and transaction accounts of non-banks*, from http://snbchf.com/monetary-fiscal-policy/snb-definitions-of-money-supply/ at 12.10.2015,15:30

[16] comp. De La Rosa, J.L., Stodder, J., 2015, p.116

References

Arestis, Philip; Sawyer, Malcolm (Editors) (2008): A Handbook of Alternative Monetary Economics, Edward Elgar, Cheltenham UK

Caverzasi, Eugenio; Godin, Antoine (2013): Stock-flow Consistent Modeling through the Ages, Working Paper No. 745, Levy Economics Institute, Annandale-on-Hudson, NY

Douthwaite, Richard J. (1999): The Ecology of Money; Totnes, Green Books

de la Rosa, Joseph L.; Stodder, James (2015): On Velocity in Several Complementary Currencies, International Journal of Community Currency Research, Volume 19 (D) 114-127, www.ijccr.net

Eucken, Walter (1941): Die Grundlagen der Nationalökonomie, Gustav Fischer, Jena

Fisher, Irving (1912): The Purchasing Power of Money: Its Determination and Relation to Credit, Interest and Crisis, New York

Greco jr, Thomas H. (2009): The End of Money and the Future of Civilization; 280, Chelsea Green Publishing, White River Junction, Vt.

Greco jr, Thomas H. (2013): Taking Moneyless Exchange to Scale: Measuring and Maintaining the Health of a Credit Clearing System; International Journal of Community Currency Research 17 (A) 19-25, www.ijccr.net

Huber, Lucas and Martignoni, Jens (2013): Improving complementary currency interchange by a regional hub-solution, International Journal of Community Currency Research, Volume 17 (A) 1-7, www.ijccr.net

Kennedy, Margrit; Lietaer, Bernard; Rogers, John (2012): People Money – The promise of regional currencies; Triarchy Press, Devon UK

Keynes, John Maynard (1989): Vorschläge für eine Internationale Clearing-Union, in: Leber, S., Wesen und Funktion des Geldes, S. 323-353, Dornach.

Keynes, John Maynard ((2006): Allgemeine Theorie der Beschäftigung, des Zinses und des Geldes; zehnte, verbesserte Auflage, Duncker&Humblot, Berlin

Lietaer, Bernard; Arnsperger, Christian; Goerner, Sally; Brunnhuber, Stefan (2012): Money and Sustainability – The Missing Link, Report from the Club of Rome, Triarchy Press, Devon UK

Dunne, Jacqui; Lietaer, Bernard (2013): Rethinking Money – How new currencies turn scarcity into prosperity; Berrett-Koehler Publishers; San Francisco

Hughton Budd, Christopher (2015): D3.2 Training Material on Accounting – Accounting, Social Innovation and Finance – Teaching financial literacy using double entry bookkeeping, CRESSI working papers No.15/2015, Oxford

Martignoni, Jens (2012): A new approach to a typology of complementary currencies; International Journal of Community Currency Research, Volume 16 (A) 1-17, www.ijccr.net

Martignoni, J., (2015): Cooperation and Intertrade between Community Currencies : From fundamentals to rule-making and clearing systems, including a case study of the Zurich Area, Switzerland, International Journal of Community Currency Research 19 (D) 137-151 www.ijccr.net

Paul, Axel T. (2012); Die Gesellschaft des Geldes – Entwurf einer monetären Theorie der Moderne, Springer VS, Wiesbaden

Paulsen, Andreas (1954): Neue Wirtschaftslehre, Verlag Franz Vahlen, Berlin

Smith, Adam (2007): An Inquiry into the Nature and Causes of the Wealth of Nations, (first published 1776) MetaLibri http://metalibri.incubadora.fapesp.br Amsterdam

Stuetzel, Wolfgang (1978): Volkswirtschaftliche Saldenmechanik, Mohr Siebeck, Tuebingen

Tobin, James (1981): Money And Finance In The Macro-Economic Process, Nobel Memorial Lecture, 8. December 1981, Yale University, http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1981/tobin-lecture.pdf